

Environmental Mathematics and Statistics

WORKSHOP

18 July 2000, Institute of Physics, London

Background

Potential collaboration between mathematicians/statisticians and environmental scientists will be the subject of a workshop held on 18 July 2000 at the Institute of Physics, London. Organised by the Natural Environment Research Council (NERC) and the Engineering and Physical Sciences Research Council (EPSRC), the key output of the workshop will be ideas for inclusion in a proposal to NERC and EPSRC for a jointly funded programme in 'Environmental Mathematics and Statistics'.

Environmental science often throws up complex and difficult mathematical questions. These range from problems in fluid dynamics and ecological modelling, to difficulties with environmental risk assessment. Collaboration between the two communities could benefit environmental science by drawing upon mathematicians' expertise and cutting-edge knowledge, and could benefit mathematics by revealing new and challenging mathematical questions. A proposal for a programme in this area is timely.

Environmental science suffers from a shortage of skills in advanced mathematical methods. Thus, a prime objective of a thematic programme would be to train existing and upcoming researchers in the cross-disciplinary skills needed for environmental mathematics and statistics.

The workshop

Attached is a list of some areas in environmental science that need tackling and which present mathematical and statistical challenges. Thematic sessions, which may include presentations, will be formed to discuss these areas in detail and identify principle topics to be included in the bid. Possible objectives of, and framework for, a research programme will be discussed in the final plenary session. A draft agenda for the workshop is attached. To facilitate a focused and comprehensive discussion, participants are requested to submit a half-page summary of their relevant research interests and ideas. Chairmen of thematic sessions will have read these summaries and will use them to structure discussion. The summaries will also be circulated to participants.

Registration instructions

Please send a paper copy of your half-page summary to Sheila Drury, NERC, Polaris House, North Star Avenue, Swindon, SN2 1EU, **by 27 June at the latest**. Please include:

- i. your relevant research interests and ideas for research topics to be included in the proposal (please limit this to half an A4 page of 12pt);
- ii. which thematic session you wish to attend (computational fluid dynamics, ecological modelling, or environmental statistics, as described below under 'Topics for Discussion'; note that the latter two are combined in the morning thematic session in order to discuss broad modelling problems);
- iii. your name, address and contact details.

If you wish to express your views without attending the workshop, you may submit a summary and mark it at the top as *in absentia*. Places are limited; we hope we don't have to turn anyone away but places will be allocated on a first-to-register basis.

Discussion topics

I. Applications of computational fluid dynamics in environmental research

There have been significant advances in the field of computational fluid dynamics (CFD), the discipline concerned with solving numerically the governing equations of fluid flow. The accuracy and computational efficiency of numerical schemes has been improved; advection processes in fluids can be simulated without the undesirable side-effects of less sophisticated schemes; and the grids on which equations are solved can be adapted to capture efficiently small scales of motion where and when they arise. The environment presents a diverse range of possible potential applications of these advances, in the fields of atmospheric science, oceanography, hydrology, dispersion of pollution in air and water and ventilation flows in buildings, to name a few. These applications present significant challenges to mathematicians, and an opportunity for 'technology transfer' into environmental science which continues to grow in importance.

Apart from the background work necessary to implement CFD techniques in environmental applications (for instance, to accommodate a particular flow geometry or structure), there is scope for development and application in many areas, a few of which are listed below.

- ***Solving the equations more accurately.***

Many features of interest in atmospheric physics and oceanography are characterised by sharp gradients (e.g. weather fronts). From a numerical point of view, problems of simulating such features are analogous to those encountered in modelling shock waves, fluid interfaces, vortical flows and combustion driven waves. A fundamental difficulty is to resolve these features correctly. It is necessary to design new numerical schemes that are both very high order of accuracy in space and time and free from spurious oscillations. These are tasks of fundamental importance for practical applications in environmental science.

- ***Efficient discretization of the computational domain***

Environmental scientists often wish to simulate in detail flows in a specific area, but are obliged to solve equations over a much bigger area because of the influence of flows far afield. Consequently, large computational resources may be required. This problem can be alleviated by space discretisation techniques that make efficient use of the computational grid. Examples are nested grids, which give increased spatial resolution over a fixed locality, and adaptive mesh refinement, in which the grid is refined around moving features of interest in the flow. Massive savings of computer cost and memory can be achieved in this way, or, alternatively, for a given cost a much more detailed computation can be performed than on a fixed-resolution grid.

- ***Numerical schemes for source/forcing terms***

The success seen in the development of numerical methods for the differential terms in the governing equations of fluid flow has not been matched by advances in methods for algebraic source terms, which will invariably be present in practical applications. These may have geometric origin, such as those terms due to bottom elevation and breadth variation in shallow water models. Other examples of source terms include reaction/production terms (e.g. in atmospheric chemistry), which may be associated with stiff, ordinary differential equations. The correct coupling of the physics and chemistry of source terms with the differential terms is difficult to achieve with current numerical methods.

- ***Exact solutions***

Exact solutions are needed for the purpose of validating numerical methods intended for practical, more complex situations. Applied mathematicians can play a key role in designing test problems, and in producing exact solutions in close collaboration with numerical analysts and CFD workers.

II. Environmental and ecological modelling

Described below is a range of problems related to environmental and ecological processes which would benefit from either a detailed mathematical investigation, substantive statistical methodology being developed, or the novel and imaginative application of existing mathematical or statistical methods in complex applications. In some problems a combination of these three approaches is necessary. For example, climate change, environmental risk assessment, monitoring the effects of genetically modified foods, and more general changes in farming practices, are broad topical areas in which there are challenges of modelling, data collection and analysis, requiring novel mathematical and statistical procedures.

● **Broad modelling problems**

A number of common mathematical and statistical challenges arise in many areas of environmental and ecological investigation.

(i) Complex analytical and stochastic modelling

Mathematical models for environmental and ecological processes tend to be complex, and to often ignore important sources of uncertainty, whereas statistical models are generally empirical, and often ignore much that is known about the underlying physical and biological processes. Advances are required in the existing mathematical, probabilistic and statistical models to provide models that sympathetically reflect the complexity of the structure of processes and that recognise characteristics of their measurement. Often models will need to combine, in a non-trivial way, deterministic and stochastic models to represent different aspects of the process.

Almost all environmental and ecological problems are spatio-temporal. Increased activity is required in this broad field, with the modelling of the specific characteristics of particular processes providing tightly defined project opportunities. Of these problems a number involve the need to move between analyses at different scales, both in model construction and the exploratory analysis of the very large and highly structured data sets. Other commonly occurring forms of complexity which need detailed modelling include non-stationary and non-linear systems with feedback.

Modern computer-intensive procedures will often be required for the analysis of data or for presenting information about complex systems. Bayesian methods of statistical inference typically will be required to incorporate model and prediction uncertainties.

(ii) Model and structural uncertainty

Deterministic mathematical and numerical models are widely used in environmental and ecological studies, but often make strong assumptions about the structural form of the process or have a number of parameters, representing physical/biological characteristics, that have been set at arbitrary values. Statistical methods have a role to play in assessing the sensitivity to these choices. Modern computational tools allow detailed process models to be embedded into statistical inference, enabling rival models to be compared in an objective framework, and the testing of parameter-redundancy. Such approaches have applications at a wide range of levels, from global climate change models, oceanography and hydrology through to genetics and ecosystems. This is an enormously important area, both practically and for advancing science. Without a proper assessment of uncertainty the deterministic models are not likely to lead to lasting scientific progress and could lead to poor policy formulation.

For illustration, consider the example of climate change. General circulation models (GCMs) are being used to determine future trends in climate based on different human-induced scenarios. Currently, uncertainty in these predictions is measured using different starting values and comparing predictions from different GCMs. This approach leads to rather small estimates of the error bands. However, this does not reflect the true uncertainty of the predictions, as essentially the same assumptions about the model are being made in all cases. Using comparisons of model output with current climate data, and particularly comparisons of the predicted climate change with observations of the spatial finger-print of climate change, can enable the uncertainty of the GCMs to be assessed and more realistic prediction bands for future global temperature rise to be obtained.

(iii) Data collection and integration

Methods need to be devised for the efficient analysis and summary of large data sets (e.g. from satellites and numerical models), for the integration of diverse data sets from different censuses, and when processes are recorded at different spatial and temporal aggregation scales.

Issues concerning the sample design, and accounting for sampling bias, and even non-random sampling, in subsequent analysis are important. Appropriate designs for data collection need to be evaluated for efficiency and cost-effectiveness. For example, the design of a network of monitoring sites needs to balance the requirements to optimise predictions, estimate the process, with the benefits of sequential exploration of the process.

● **Ecological modelling**

The following is an illustrative list of topics requiring research at the interface between ecology, mathematics, statistics and operational research.

(i). Methods are needed for modelling ecological processes taking into account the spatial structure of environments. This includes developing techniques for modelling stochastic events acting on individuals in heterogeneous landscapes, deriving dynamical systems that incorporate essential information on spatial structure, and characterising the behaviour of the dynamical systems. Such research transcends disciplinary boundaries between stochastic processes, nonlinear dynamics and ecology, and is essentially unexplored at present.

(ii). Inferential procedures are needed for highly-structured population dynamic models, such as multi-species populations distributed in time and space. This is timely, as suitable data are now becoming available, supported by GIS, and new computational methods, such as Markov chain Monte Carlo, make analysis feasible. There is potential for better models for the movement of animals, especially as a result of fragmentation of habitat. Better procedures are needed to evaluate the effects of culling. Procedures need to be developed for investigating the interactions between individual animals and the environment.

(iii). Techniques from the area of Operational Research can be useful for solving complex optimisation problems in the field of conservation and environmental biology. An example would be the problem of designing reserves to maximise the representation of a variety of elements of biodiversity.

(iv). Novel models for distance sampling, for estimating animal abundance, and novel methods of mark-recovery-recapture, for estimating animal survival, need to be devised in response to the demands of modern methods of data collection.

● **Environmental statistics**

The following are some example problem areas in environmental statistics.

(i). Specific areas where detailed spatial and temporal modelling is required are: building models for rain-fields using data from radars and a network of rain-gauges, aggregation and disaggregation problems in hydrology, estimating trends in sea-levels through pooling regional and climate information and accounting for land-level trends, and analysing pollution dispersal and sediment movement.

(ii). Measures of environmental risk need to be improved to better reflect the uncertainty of the risk and the spatial variation and extent of the risk. For example, the risk of an event occurring at a site is typically measured by its estimated return level/period. The uncertainty in such estimates needs to be routinely evaluated; standard errors are sometimes used but predictive approaches are more informative. For spatial processes return levels are sometimes mapped over a region of interest. Current methods ignore both the uncertainty in estimates and their dependence over separate sites. However, such maps do not provide an adequate summary of the spatially integrated risk that is required by the insurance industry and for the assessment of the ecological impact of the event. Associated with this aspect is the important topic of the setting and testing of environmental standards.

(iii). A broad class of risk assessment problems involves understanding and modelling extreme values. Extreme value techniques from the 1970's and 1980's remain the most widely used in environmental studies. Use of advanced extreme value methods is required in application areas such as regional approaches in hydrology, floods in estuaries and at river confluences, dependence of river floods on catchment wetness, the analysis of

radar/satellite measured ocean wave data, air pollution assessment, and spatial modelling of extreme sea levels. For most environmental processes trends in extreme values are important; much more careful study is needed to assess their relationship to climate change and trends in the mean values of the processes.

Draft agenda

1000-1030 Registration and coffee

1030-1225 Opening presentations and discussion

Dr Alasdair Rose (EPSRC) and Mr Ian Dwyer: background (10 mins)

Keynote address: Lord Hunt, UCL (20 + 10 mins)

Professor Alan O'Neill: computational fluid dynamics (20 + 5 mins)

Professor Byron Morgan: ecological modelling (20 + 5 mins)

Professor Jonathan Tawn: environmental statistics (20 + 5 mins)

1230-1300 Parallel thematic sessions I

Computational Fluid Dynamics (rapporteur: Prof O'Neill)

Broad modelling problems (rapporteur: Profs Tawn and Morgan)

1300-1400 LUNCH

1400-1500 Parallel thematic sessions II

Computational Fluid Dynamics (continued)

Ecological modelling (rapporteur: Prof Morgan)

Environmental statistics (rapporteur: Prof Tawn)

1500-1630 Plenary discussion (Chaired by Prof O'Neill)

Presentations from Thematic sessions (3 x 5 minutes)

Open discussion on science plans, programme objectives and format (1 hour)

Concluding remarks and identification of actions (5 mins from Mr Rose, 10 mins from Prof O'Neill)